What is a base?

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The biological fact that we have ten fingers has made it almost inevitable that we should count in tens since the use of fingers is a time honoured way of counting, and it is even a neurological fact that the part of our brain that controls finger movement is very close to the part we use for mathematics, in particular for counting. A teacher telling a child not to use fingers is in effect saying "Don't do mathematics!" In any case, we "base" our counting on the number ten, probably thinking of our fingers in racial memory! And when we have ten lots of tens, we come to the "next order of magnitude" and we go on like that accumulating powers of ten, so we can express ever larger numbers in figures.

Had we evolved with a different number of fingers, or had we not counted thumbs in with our other fingers, we would have "based" our counting on a different "base number".

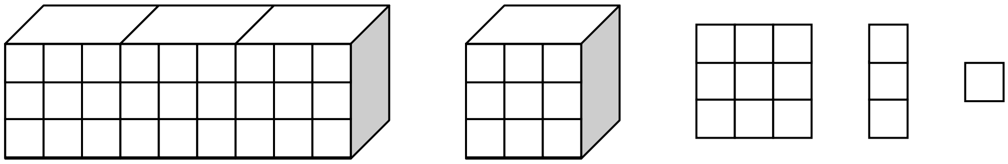
The position of the written digits in a written number tells us whether they are counting singles or tens or hundreds or higher powers. This is why our system of numbering, introduced in the middle ages by Arabs, is called the place value system.

My contention has been, that in order to fully understand how the system works, we have to understand the concept of power. Further, in order to understand any mathematical concept, we must understand how the variables that it contains are related to each other. The concept of power has two variables: the base and the exponent. In order to get to understand the concept of power, it seems reasonable that we should have some experience of varying both variables. In school, when young children learn how to write numbers, they use the base ten exclusively and they only use the exponents zero and one (namely denoting units and tens) , since for some time they do not go beyond two digit numbers. So neither the base nor the exponent are varied, and it is a small wonder that children have trouble in understanding the place value convention.

So I have been suggesting, for the past half century, that different bases be used at the start, and to facilitate understanding of what is going on, physical materials embodying the powers of various bases should be made available to children. Such a system is a set of multibase blocks, which I introduced in England, Italy and Hungary in the 1950's. Educators today use the "multibase blocks", but most of them only use the base ten, yet they call the set "multibase". These educators miss the point of the material entirely. Teachers who have used the material from the very start swear by it and would never go back to "base ten only" teaching. Another advantage of using various bases is the ease with which certain algebraic identities can be constructed and properly understood.

I will just give a few examples of how the material can be used, and finish with an amusing story about one way it is fun to introduce "bases"

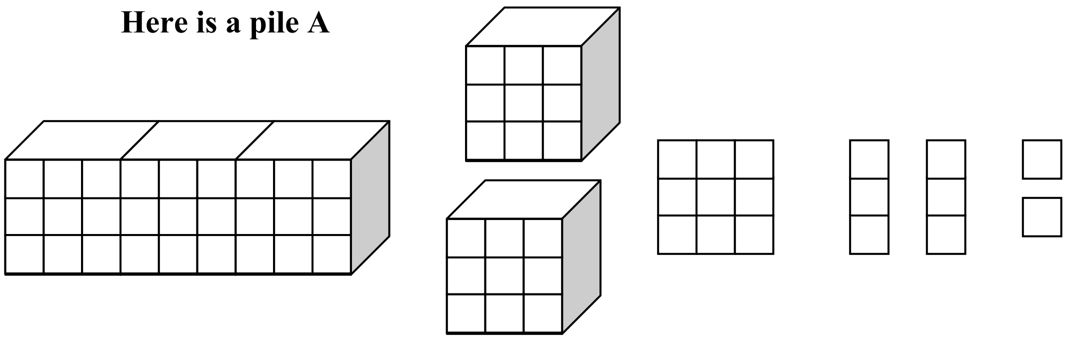
Here is one way of representing the base of three



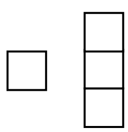
3^4 3^3 3^2 3^1 3^0 (units)

Here is how a "long division" would go:

Here is a pile A



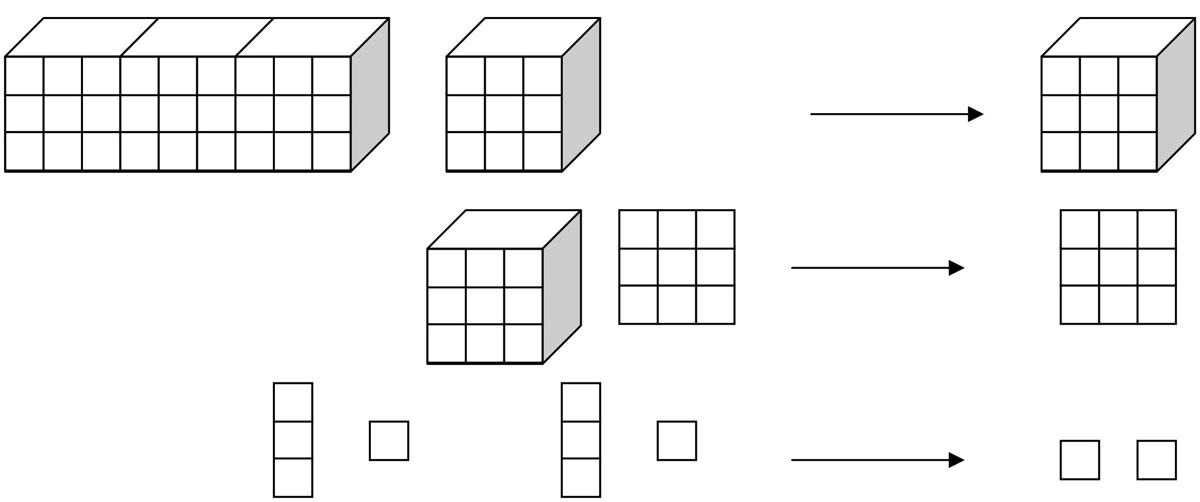
And here is a pile B



How many times can you find piles like B
In pile A?

In fact make a pile in which there are just as many units as there are piles like B in pile A.

Pile A | Pile of "units" required



The above could be written in symbols as follows:

1 2 1 2 2	divided by 1 1	
1 1 0 0 0		1 0 0 0
1 1 0 0		1 0 0
2 2		2
	Total =	1 1 0 2

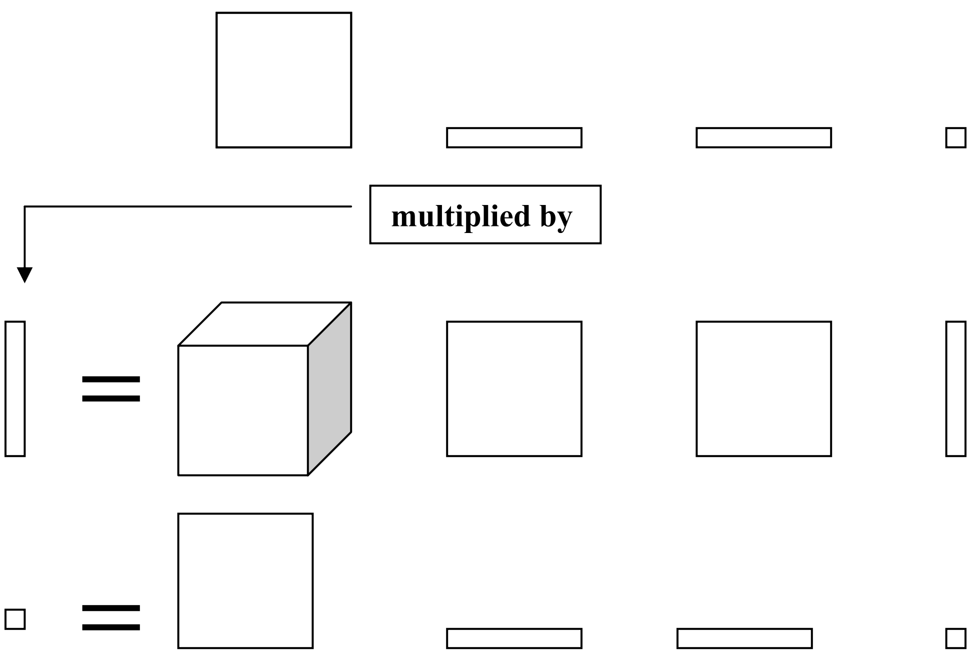
Do not forget that 1 2 1 2 2 means

1 (eightyone) + 2 (twentysevens) + 1 (nine) + 2 (threes) + 2 (ones)

and that 1 1 means 1 (three) + 1 (one)

You just have to look at the ratio of what is on the right to what is on the left, all the ratios are (1 : 1 1) or in usual terms (1 : 4)

In some situations we obtain the same formal result in operations such as multiplication, whatever base number we use. Here is an example:



Or algebraically $(b^2 + 2b + 1)(b + 1) = b^3 + 3b^2 + 3b + 1$

In some of the "conversational mathematics" that you will see, base eight is used, since the shipwrecked children, Bruce and Alice, find themselves in Eightland, where everything is done in eights, and of course counting is in base eight. So you will have to remember that when you see the "number" 1 2, it means one eight and two ones, and 1 0 3 will mean sixty-seven, namely one sixty-four and three ones and so on.

Here is an amusing situation described in verse. You can find many more of a similar kind in my book of poems "Calls from the Past".

Martian counting.

In the land of the Swiss I once went to a school
With some children some maths games to play.
A colleague who came with me asked: "What's your tool?"
"All they need is their brains for the fray!"

When I entered the class I said I was from Space,
From the Red Planet I'd just arrived.
There was wondrous amazement on everyone's face,
But none thought that I had them beguiled!

I said: "I want to count all the children in here,
But on Mars we count only to three!"
"You will never be able to count us I fear"
Said a small boy as he laughed with much glee.

"Yes, I can!", I replied, saying: "Ein, zwei und drei!"
With each word touching one other child.
Thus I went through the class and a child then did say:
"Martian counting does seem very wild!"

"Do I know the right number of children in here?"
I did ask with a questioning air.
"You don't know all the times you have said ein, zwei, drei!"
"Then we'll count ein zwei dreis to be fair!"

When I got to the third ein zwei drei I did stop,
As a Martian I could count no more!
"If you don't want this thing to become quite a flop,
Ein zwei dreis said three times we must score!"

So we went on to count all the triple ein zwei dreis,
But of these we could find only two!

So with two triple ones and with two ein zwei dreis,
There was one still uncounted, it's true!

"Do I know now in Martian the all final score?"
"Yes, you do, if you count as on Mars,
With two nines and two threes and another child more,
You can send this e-mail to the stars:

In a school in Grade Two in the Zurich canton
There are two two one children in class.
But you never would guess all the things that went on,
They can count more than three! What a farce!"