APPENDIX II

THE MISSING TENNIS GAMES

Here is one of the missing tennis games that Bruce secretly smuggled back into THREELAND at the risk of being sent to prison:

The ball falls in	0	1	2	3	4	5	6	7
served by 0	0	1	2	3	4	5	6	7
served by 1	1	0	3	2	5	4	7	6
served by 2	2	3	4	5	6	7	0	1
served by 3	3	2	5	4	7	6	1	0
served by 4	4	5	6	7	0	1	2	3
served by 5	5	4	7	6	1	0	3	2
served by 6	6	7	0	1	2	3	4	5
served by 7	7	6	1	0	3	2	5	4

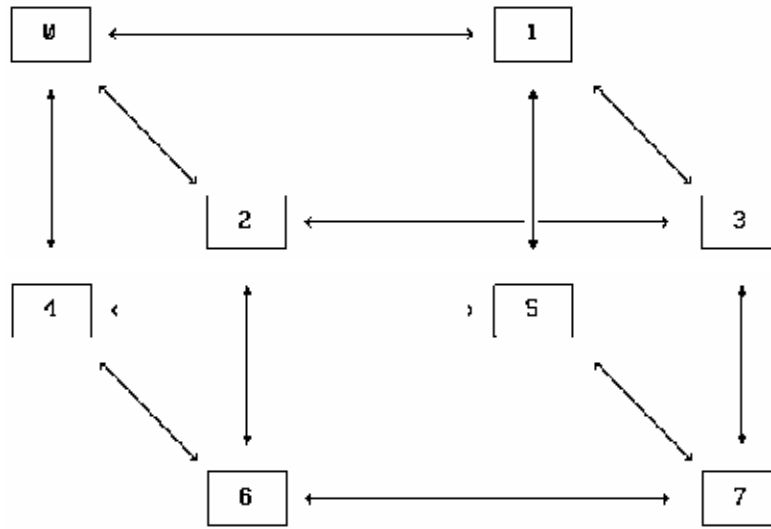Notice how the even numbers are placed in this table.

Notice also the two by two squares in which only two different numbers occur whose difference is exactly one. These squares are placed in the table according to a strict rule. Can you see it?
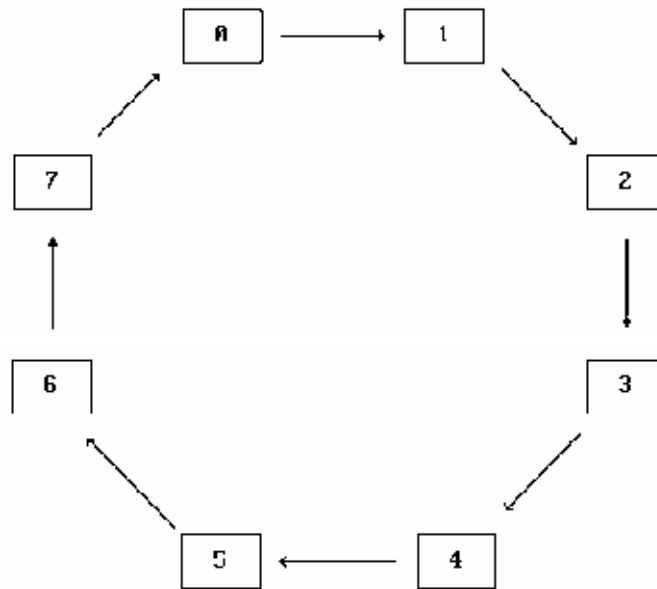
Here is another missing one

The ball falls in	0	1	2	3	4	5	6	7
served by 0	0	1	2	3	4	5	6	7
served by 1	1	4	3	6	5	0	7	2
served by 2	2	7	4	1	6	3	0	5
served by 3	3	2	5	4	7	6	1	0
served by 4	4	5	6	7	0	1	2	3
served by 5	5	0	7	2	1	4	3	6
served by 6	6	3	0	5	2	7	4	1
served by 7	7	6	1	0	3	2	5	4

Try to find some fun aspects of this table.

Where are the pairs of numbers that differ by 4 ?

Here is a way of remembering the rules of the "two by two by two" tennis game:

```
 ┌───┐                          ┌───┐
 │ 0 │ <──────────────────────> │ 1 │
 └───┘                          └───┘
   │  ↖                    ↑   ↗   │
   │    ↘     ┌───┐        │  ↗    │
   │      ↘   │ 2 │ <───── │ ──────> ┌───┐
   ↓         └───┘         │        │ 3 │
                           │        └───┘
 ┌───┐                  ┌───┐          │
 │ 4 │ <            >   │ 5 │          │
 └───┘      ↑           └───┘   ↗      ↑
   │  ↘     │                  ↗       │
   │    ↘ ┌───┐              ↗         ↓
   ↓      │ 6 │ <──────────────────> ┌───┐
         └───┘                       │ 7 │
                                     └───┘
```

Here is another way of remembering the 8-game that goes round the courts clockwise:

```
        ┌───┐        ┌───┐
        │ 0 │ ─────> │ 1 │
        └───┘        └───┘
      ↗                    ↘
 ┌───┐                      ┌───┐
 │ 7 │                      │ 2 │
 └───┘                      └───┘
   ↑                          │
                              ↓
 ┌───┐                      ┌───┐
 │ 6 │                      │ 3 │
 └───┘                      └───┘
      ↖                    ↙
        ┌───┐        ┌───┐
        │ 5 │ <───── │ 4 │
        └───┘        └───┘
```

Below you will find the "maps" of two more of the tennis games. Try to work out which is which!
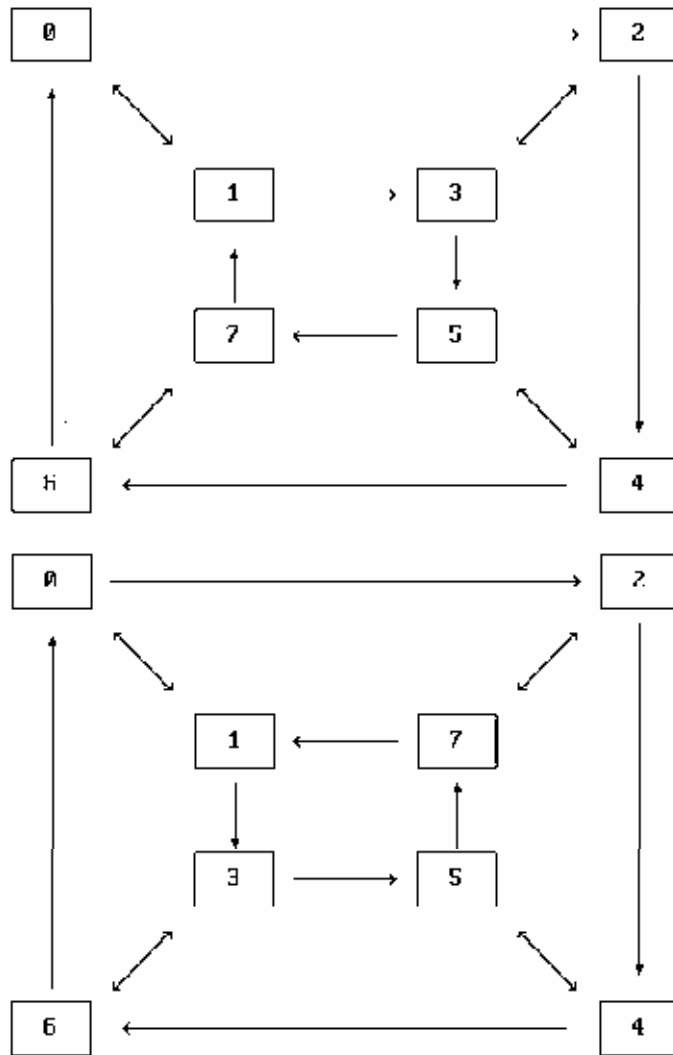
This game came to be called the SQUARE GAME, because there is a way of writing these numbers on a square piece of wood, four numbers on each side, so that by combining flipping and turning the square, the numbers should turn up in a regular fashion. The odd numbers should go on one side and the even ones on the other. Try to find out how that works.

Here is the "map" of the hard one that Karo found difficult to learn:
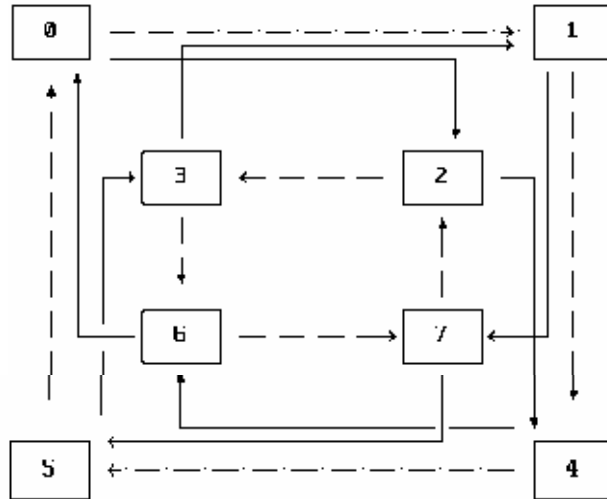
Each rule is made up of a certain number of steps you can make from number to number, following the broken and/or the unbroken lines. For example:

The 1-rule = take one step along the broken lines

The 2-rule = take one step along the unbroken lines

The 3-rule = take one step along an unbroken line, followed by one step along a broken line (but NOT the other way round!).

Do you get the 4-rule by doing any other rule twice one after the other? Try to check.

Try to identify all the other rules from the map, and compare them with the table and see if they correspond!

Many years ago someone from Europe came to study the Ruritanian games, and he was rather taken by this one. For some reason only known to himself, he called it the

QUATERNION GAME

or the Quat-game, for short, and this name has stuck, although nobody knows why the traveller should have called it by that name.