"All are" and "There is at least one"
(Games to amuse you)

The games and puzzles in this section are to do with using the terms “all”, “not all”, “there is at least one”, “there isn’t even one” and such like. Such phrases are said to quantify statements.

Let us think of a UNIVERSE of children, namely all the children in a school, or at a campsite, or in an apartment block. Probably the simplest statement to make about a child is to state the sex of the child, namely :whether boy or girl. Imagine a certain set of children, taken out of our UNIVERSE, then each of the following statements can be verified to be either true or false:

(i) **all the children are boys**
(ii) **not all the children are girls**
(iii) **there is at least one girl**
(iv) **there is not even one girl.**

Let us also allow ourselves to speak about non-boys or about non-girls. Clearly non-boy and girl are interchangeable, and so are non-girl and boy.

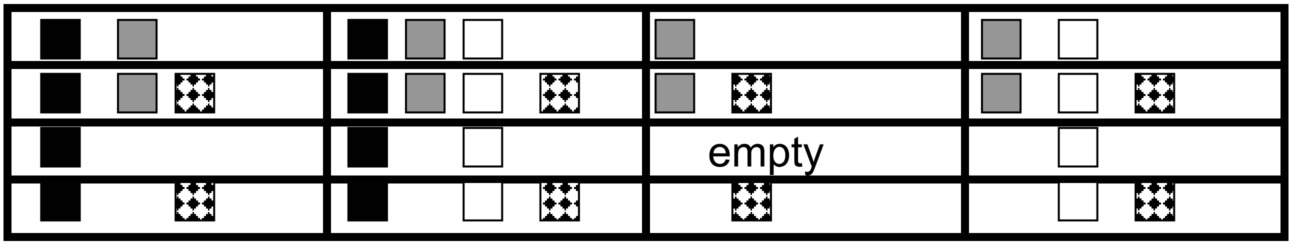
I am suggesting this way of referring to “children who are not boys” because sentences such as

“All the children are not boys”

are ambiguous, as it is not clear whether it is meant that not every child is a boy, namely that there is at least one girl in the set, or whether it is meant that all of them are children who are not boys. It is in fact not clear whether the “not” negates the “all” or the “boys”

There are sixteen statements that we can make which are the following :

all are boys	not all are boys	all are girls	not all are girls
all are non-boys	not all are non-boys	all are non-girls	not all are non-girls
there is a boy	there is no boy	there is a girl	there is no girl
there is a non-boy	there is no non-boy	there is a non-girl	there is no non-girl



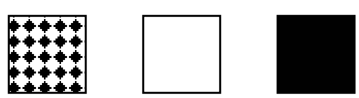
In the second table we can find a way of embodying in a pictorial way all the sixteen statements.
Black means that the word boy or boys occurs.
Grey means that the word all occurs.
White means that not or no occurs.
Patterned means that non occurs.

It should be noted that the absence of a symbol will also give us information. For example lack of black obviously means girl or girls.

GAME 1.

One obvious game can be played by players challenging each other to provide the correct set of symbolic objects, given a statement, or conversely, given a set of symbolic objects, the player in question must give the corresponding sentence.

For example take the set of objects



which means that the statement must have the word "boy". The lack of gray means that "there is" must be in the statement, white means "not" or "no" so in our case it must be "there is no". Patterned means that the word "non" is there, so it must be about "non-boys". So the sentence is

"There is no non-boy"

GAME 2.

In this game players will try to identify the sets for which a particular statement is true. There will be three sets

A set consisting of boys only, the set B.
A set consisting of girls only, the set G.
A set with both boys and girls, the set M.

For example taking the statement

“There is a girl”

the player should point at the sets M and G.

The game could be played the other way round as well. One player points at a set, and the other player will have to reply with as many statements as possible which are true for the set in question. A score of one should be given for a statement which is true for the set in question, and a score of minus one for a statement which is false for the set in question.

A more challenging version could be played by one player pointing to two sets. The other player should either state that there is no sentence true for both sentences, or state as many sentences as possible which are true for both sentences.

For example if B and G are chosen, there will be no statements true for both sets. But if B and M are chosen then there will be several. In fact the statements

“there is a boy”, “there is a non-girl”,

“not all are girls”, "not all are non-boys”

are all true for both sets B and M.

GAME 3

One player selects any one of the sixteen statements.

The other player, having decided for which set or sets the statement is true, must find the other three statements that are true for the same sets.

For example if a player proposes

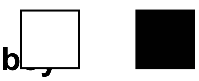
"There is a boy"

then the other statements must be found which are true for B and M. These are, in this case :

"There is a non-girl", "Not all are girls"

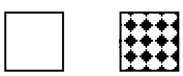
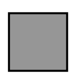
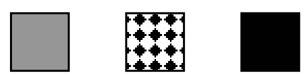
"Not all are non-boys"

Instead of selecting a statement, the game can also be played by selecting a set of symbolic objects. The three other sets of objects must then be produced which correspond to the three statements which are true at the same time as the statement corresponding to the set of symbolic objects chosen.
For example, suppose that the set



is chosen "there is no boy"

Then the other three sets will be

 ,  

"there is no non-girl" , "all are girls" "all are non- boys"

where any of the sets can be changed into one of the others by one of the following three changes :
change grey white patterned,
change black grey white
change patterned black

change meaning removing if it is there, putting it in when it is not there.

Statements that are true for the same sets and false for the same sets are called EQUIVALENT.

The purpose of the game just described is to encourage players to find out how they can tell whether two statements are equivalent to each other.

Game 4.

In this game two statements are chosen by one player, and the other player must say :

either that the two statements are equivalent
or that one of them is the negation of the other,
or that neither of the above is the case.

For the first to be the case the two statements must be true for the same sets and false for the same sets. For the second to be the case, the set or sets for which one is true must be false for the other.
For the third to be the case, there must be an "overlap" between the set or sets for which one statement is true and the set or sets for which the other statement is true.

After playing this game for a while players will probably come to know some "tricks" for deciding the issue, so they do not have to check the corresponding sets every time.

We can consider each statement to be made up of four parts:

The first tells us whether there is a NOT or NO.
The second part tells us whether there is an "all" or a "there is"
The third part tells us whether there is a "non"
The fourth part tells us whether it is about boys or girls.
Let us write N for "not" or "no",
Let us write A for "all" and E for "there is"
Let us write N for "non"
Let us write b for boy and g for girl.

In this way our statements will become much shorter, more concise, and more easily comparable to each other.

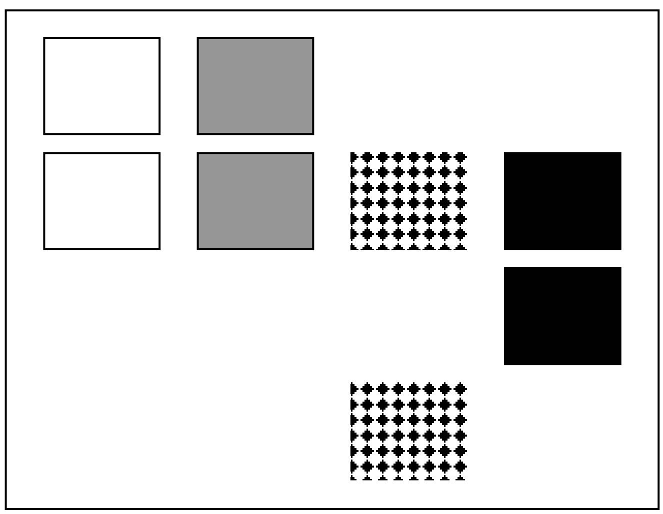
If we do not write an N, we shall leave a blank space. For example

“not all are girls”

will be written as

	N	A		g
equivalent to	N	A	N	b
and to		E		b
and to		E	N	g

or in "colors" :



To pass from one row of symbols to another we have the same rules as before, namely the changes

white-grey-black, white-grey-patterned, patterned-black

which correspond to changing what is written in the first, second, third and fourth cells respectively:

1 2 4, 1 2 3, 3 4

where the numbers indicate the cells to be changed, counting from left to right.

It might be worth noting that any two of the above changes done one after the other, can be replaced by the remaining change. For example changing the first, second and fourth cells and then the third and fourth cells, we will have changed the first, the second and the third cells.

It is not hard to see that in order to negate one of our statements we must change the cells in one of the following four ways :

1, 1 3 4, 2 3, 2 4

or if we are using the symbolic objects, we must make one of the following changes :

white, white-patterned-black, grey-patterned, grey-black

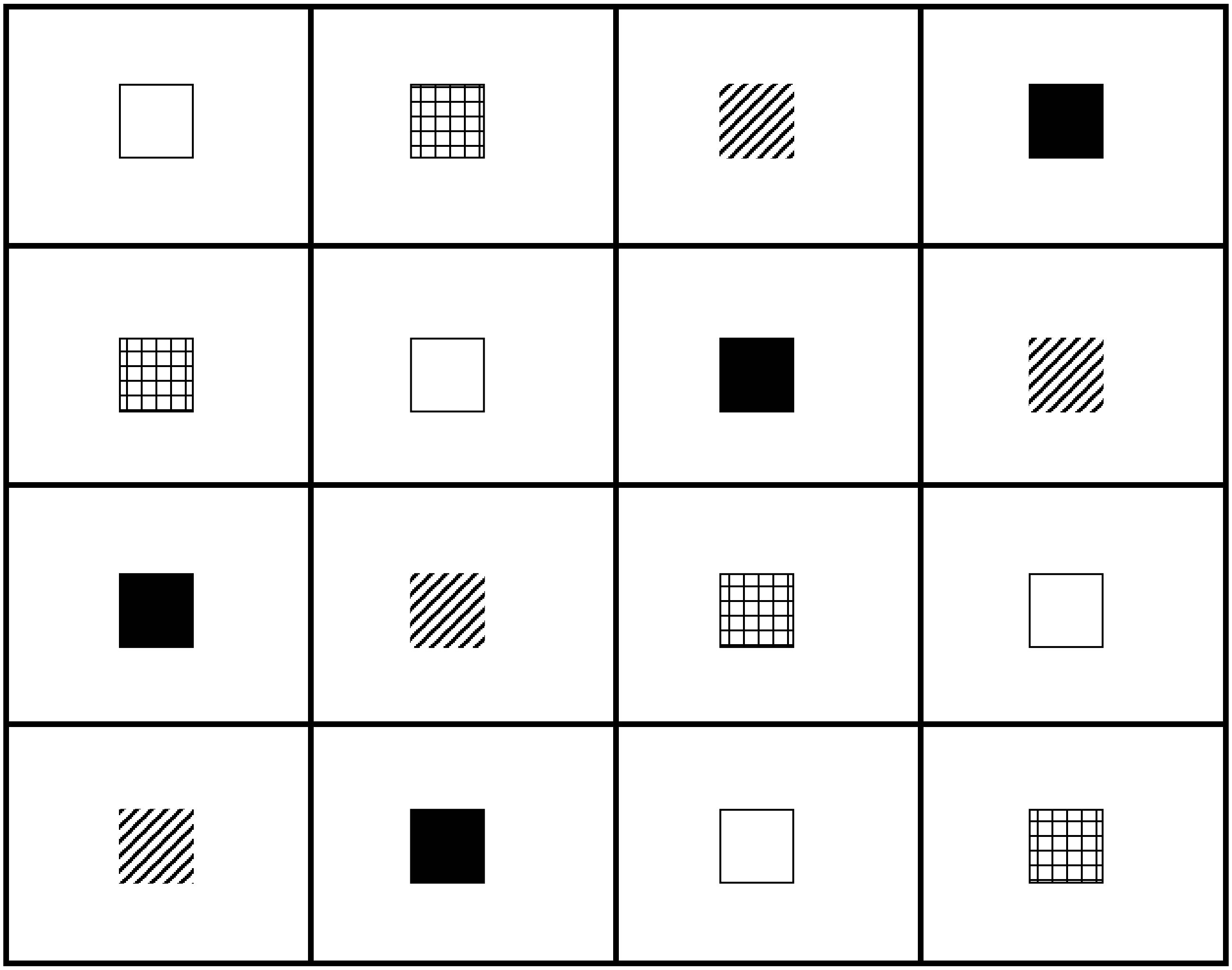
If we do so, we shall always get a corresponding statement which is true now when it was false before and false now when it was true before, for any of our three sets of children.

Game 5

In this game players are asked to place their sixteen statements in a four by four "square", so that the sets of four statements that are equivalent to each other should be symmetrically placed. It is left to the players' aesthetic sense to decide what "symmetrical" means. Here is one attempt at a "solution" of this rather purposely vague problem

N A N b	N A b	N E N b	N E b
N A N g	N A g	N E N g	N E g
A N b	A b	E N b	E b
A N g	A g	E N g	E g

Below you can see how the sets of four equivalent statements are situated in the table. If you turned the symbol table round through ninety degrees, each symbol is simply replaced by another symbol, the cells of the same symbol forming shapes like T's, with the lower part of the T taking up one half of a diagonal.



Cells of the same kind indicate corresponding statements that are equivalent to each other. These patterns have no relation to the codes used for representing the various parts of our statements.

Double quantifiers.

It is possible to have two quantifiers in the same statement. For example you might make some sets of logic blocks and then try to check whether statements of the kind

"There is a set in which all the blocks are red"

are true or false for your arrangement of sets. Or you might want to test statements such as

"In all the sets there is a red block"

and so on.

We are "quantifying" the sets as well as the blocks. In the last statement "all" refers to the sets, and "there is" refers to the blocks.

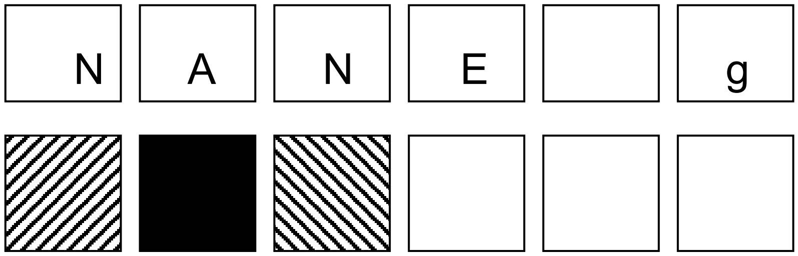
We can again have red blocks and blue blocks and non-red blocks and non-blue ones, and we can talk about "not all" and "there is no", as referred either to sets or to blocks. So there are three places in the statement where we can place a "not" or a "no" or a "non", there are two places where we can place quantifiers (quantifying either sets or blocks), and we have to decide whether the word red or the word blue is going into the statement. So each statement will now consist of six parts instead of four, and so if we want to play the "symbolic pieces game", we shall need six different symbols.

Let us think of a town in which there are seven schools.
In school B there are only boys' classes.
In school G there are only girls' classes.
In school M there are only mixed classes.
In school BG there are boys' classes and girls' classes.
In school BM there are boys' classes and mixed classes.
In school GM there are girls' classes and mixed classes.
In school BGM there are boys' classes, girls' classes and mixed classes.

Now we can talk about the classes (saying "all" or "not all" or "there is" or "there is no") or we can be talking about the children. Lastly we can say whether we want the word "boy" or the word "girl" in the statement. For example we could make a statement such as

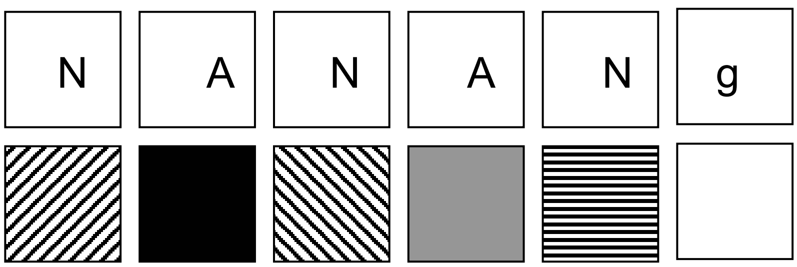
"In not all the classes is it so that there is no girl"

which we could write in our abbreviated form as



using "ascending strokes" for the “not” that refers to the classes and black for the “all” that refers to the classes, while using "descending strokes" for the “not” that refers to the children and grey for the “all” that refers to the children, while "horizontal strokes" refer to “non” and dots are used if we wish to have the word “boy”. In the contrary cases we shall either just use nothing, or if we use an orderred set of six symbols, an empty square.

Here is another example :



“In not all the classes are not all the children non-girls”

There are altogether 64 statements we can construct, and we can use either the “abbreviated writing” or the "symbolic pieces" system, as shown above.

A deck of 64 cards can be made, on one side we should have a statement, on the reverse side the list of the schools for which the statement is true.

Game 6.

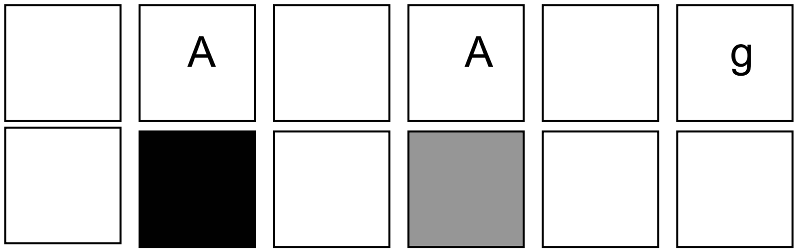
One player picks one of the 64 statements at random by drawing a card from the deck.

The other player must say exactly for which school or schools the statement is true.

Then this player should also find the other seven statements each of which is true for the very same school or schools as the statement first chosen.

Here is an example.

Suppose we start with the statement



"In all the classes all the children are girls"

We only have one school for which the above is true, it is school G. The other statements which are true only for school G are

A A N b, N E E b, N E E N g, A N E b,

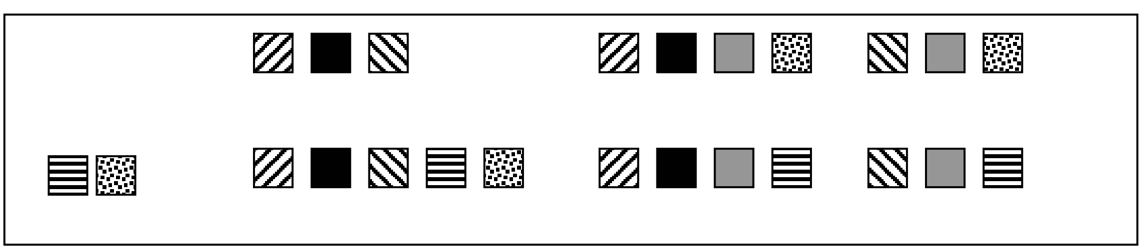
A N E N g, N E N A g, N E N A N b

The changes in the six cells that lead from a statement to an equivalent statement are the following (the cells are numbered from left to right):

No change	**1 2 3**	**1 2 4 6**	**3 4 6**
5 6	**1 2 3 5 6**	**1 2 4 5**	**3 4 5**

Any two of these changes carried out one after the other, will result in one of the changes already present in the above table. Combining the changes in either order will result in the same "equivalent" change.

The "symbolic pieces" changes that correspond are



To get the rules for the negation of a statement, all we have to do is to place an N in front if there is not one there, or leave the N out, if there is one there. In the case of the "symbolic pieces", the "moves" will be the same as the above, with the "ascending stripes" added if absent, or removed if present. So each statement will be able to be negated in eight different ways. Naturally each of these negations will be equivalent to each other.

Here is a table of all the sixty four statements:

A A b	N EN A b	N E E g	A N E g
A A N g	N E N A N g	N E E N b	A N E N b
N A A b	E N A b	E E g	N AN E g
N A A N g	E N A N g	E E N b	N A N E N b
A E b	NE NE b	N E A g	AN A g
A E N g	N E N E N g	N E A N b	A N A N b
N A E b	EN E b	E A g	NA NA g
N A E N g	E N E N g	E A N b	N A N A N b
E A b	NA N A b	NA E g	E NE g
E A N g	N A N A N g	N A E N b	E N E N b
N E A b	AN A b	A E g	NE N E g
N E A N g	A N A N g	A E N b	N E N E N b
E E b	NAN E b	N A A g	E N A g
E E N g	N A N E N g	N A A N b	E N A N b
N E E b	AN E b	A A g	NE N A g
N E E N g	A N E N g	A A N b	N E N A N b

Rows 1 and 2 have statements equivalent to each other, but rows 3 and 4 have the negations of the statements in the first two rows. This continues in the remaining sets of four rows, rows 5 and 6 having equivalent statements, and rows 7 and 8 providing their negations, and so it continues.

The first two rows refer to school B only. Rows 3 and 4 refer to any school except school B. Rows 5 and 6 refer to one of the schools B, M or BM. Rows 7 and 8 refer to any of the schools G, BG, MG, BGM. Rows 9 and 10 refer to any one of the schools B, BG, BM, BGM. Rows 11 and 12 refer to any of the schools G, M, GM, while rows 13 and 14 refer any school which is not school G, the last two rows referring to school G alone.

In the deck of cards the schools for which each statement is true is written on the back of the card on which that statement is written.

The game can be played by drawing a card and looking at the statement, the player having to state all the schools for which the statement is true. The response is checked by looking at the back of the card. Or conversely, a player might look at the list of schools on one side and then enumerate all the eight statements which are true for each one of that particular set of schools.

If desired, the "symbolic pieces" coding could be brought into the game as well. A set of pieces can be given and both the corresponding statement and the set of schools for which that statement is true, must be given in response.

GAME 7

Deductions from one statement.

Sometimes it is possible to say that whenever a certain statement is true, then a certain other statement is also true. We say that the first statement IMPLIES the second statement, or that we can from the first statement DEDUCE the second statement.

Given any two of our 64 statements, how can we tell whether one of them can be deduced from the other ?

This is done quite simply by looking at the sets for which each statement is true.

Let us call the SET of SETS for which a statement is true the TRUTH-SET of that statement.

Given two statements A and B, if the truth-set of A is a subset of the truth-set of B or if it is identical with the truth-set of B, then from the statement A we can DEDUCE the statement B.

Here are some examples :

A E N b G, M, GM	→	E E N b G, M, BG, BM, GM, BGM
A E b B, M, BM	→	N A A g B, M, BG, BM, GM, BGM
A A g G		E A g G, BG, GM, BGM

If the statement on the left is true, we must be thinking of one of the schools written underneath that particular statement. The corresponding statement on the right also has those same sets written underneath it (along with some others as well), so our statement on the right must also be true for any set belonging to the truth-set of the left hand statement, since those sets are also members of the truth-set of the right hand statement. So we can DEDUCE each right hand statement from the one standing on its left, but of course NOT the other way round. In order to be able to deduce either one from the other, the two truth-sets must be the same, in other words the statements in question must be EQUIVALENT to each other.

Our 64 statements are split into eight sets of eight statements, and within each set of eight, all the statements are equivalent to each other as they have the same truth-set. So each of the eight can be DEDUCED from any one of the other seven.

Game 8

Deductions from two statements.

In order to deduce a third statement from two given statements, we must assure ourselves that the third statement will be true whenever both our initial statements are true. This will clearly be the case if the common part of the truth-sets of the first two statements is either identical with or is a subset of the truth-set of the third statement which we are wishing to deduce from the first two.

The first two statements will be called the PREMISES, the third one, if it can be deduced from the first two, will be called the CONCLUSION.

For example let us take the following two premises along with their truth-sets

E A g	E A b
G, BG, GM, BGM	B, BG, BM, BGM

If we now consider the statement

E E g as a possible candidate for a conclusion,, we are in business because the truth-set of this statement is

G, M, BG, BM, GM, BGM

The common part of the truth-sets of our premises is { BG, BGM }, which is clearly a subset of the truth-set of our proposed conclusion statement.

So from E A g and E A b we can conclude

E E g

but we can also conclude E E b, whose truth-set
is B, M, BG, BM, GM, BGM
which also includes BG and BGM.

So what we are saying, "in the vernacular", is that in a school in which there is an all girls' class as well as an all boys' class, there is a class with a girl in it and there is a class with a boy in it! Not a tremendous discovery, yet common sense is there to check that what we are doing in our formal work corresponds to the reality which it is meant to represent.

It would now be relatively easy, although a little complicated, to extend the quantifying to using three quantifiers in the same sentence. We could have a whole lot of towns in which there are different kinds of assortments of schools. You would have 256 statements to sort out, and you would need eight symbols for the coding of the statements, and you would need goodness knows how many different towns in which different assortments of the seven kinds of schools would be present. It is perhaps time to take a look in a book on Logic and learn in the "old fashioned" way how such statements, with so many quantifiers, hang together! With the experiences gathered from these pages, perhaps the mathematical hieroglyphics in a book on Logic might begin to make sense! But let me end this section with a true story about a little girl who went shopping with her mother (actually, it was my wife, as expounded to me by her mother!):

As mother and child were nearly home from some extravagant shopping at Peter Jones's, the mother said to her little girl:

"Don't tell your father that we've been to Peter Jones's!"

As they entered the house, and were greeted by the Man of the House, the little girls blurted out:

"Daddy! We haven't been to Peter Jones"s!"

"I expect that is just where you've been!", replied the little girl's father.

What had gone wrong? The little girl's mother meant the NOT to refer to TELL, meaning DON'T TELL, but the little girl put the NOT to refer to HAVE BEEN, making it "WE HAVEN'T BEEN". The mother meant

DON'T TELL THAT WE'VE BEEN

but her little girl understood it as

TELL THAT WE HAVEN'T BEEN.

There were two predicates, telling and having been, and the little girl put the NOT in front of the wrong predicate, thus creating matrimonial disharmony!

The moral of the story is: Be careful to/ make sure that any NOT you say is referred to the desired predicate. To avoid the ambiguity in a statement such as

"Not all the children are present" or "All the children are not present"

which could be interpreted linguistically as meaning the same thing, but to make sure that the "not" refers to "present" and not to "all" you might need to reformulate the second sentence into "All the children are absent", where "absent" makes sure that you mean "not present" and not "not all" !

For more information about the work of Zoltan P. Dienes, see the web site at

http://www.zoltandienes.com