Chapter IV
Ruritanian Games

The next day was a school day again. The children had lessons in History, Geography, Arithmetic and Grammar. The Grammar lesson was in part about English grammar, and in part about the Threelandish form of the Ruritanian language, which Bruce and Alice found quite interesting. They thought it was a good idea to learn something of the language of their newfound friends, as they would then not feel so left out when the children talked to each other in their own language.

"Des dozam robni kom grammatik tralandika", said the teacher to the children to open the lesson.

"What does that mean?", whispered Alice to Alo, who was sitting next to her.

"It means that we must now work on the Threelandish grammar", replied Alo as softly as he knew how.

"Si ragi stra ne kapan, potan redni livo por aritmetik", said the teacher.

"That means, if you don't understand, you can read your book on Arithmetic!", said Alo to Alice, but so that Bruce could hear it too.

"Vam kapni tralandik!", said Bruce bravely, looking at the teacher.

"Bona, bona!", replied the teacher.

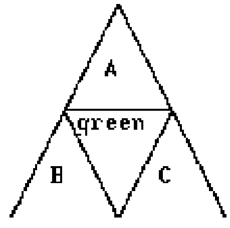
Bruce had been secretly studying some Threelandish, and had managed to say "We want to understand Threelandish" in Threelandish! The teacher replied by saying "Good, good!"

So both Bruce and Alice decided to listen intently and so to try to pick up some of this interesting local language. The other children in the class were very impressed and promised to help them in their efforts.

The lessons only lasted until midday, as after lunch it was time to play games on the playing fields. Unta had said that Bruce and Alice would not find it hard to learn the games, saying that they were fun to play.

After lunch, all the children went to the playing fields. This was a very large area and a lot of it was taken up by enclosures, the fences around these enclosures being quite high, at least two quartas high, and they were all made in the shape of triangles. Bruce and Alice realized that this was in keeping with the Threelanders' preoccupation with the number three.

Each of the triangular enclosures was divided into one-one smaller triangles, one at each corner and one in the middle. The middle one was made into a lawn, whereas the corner ones were asphalted. The middle triangle was separated from the corner ones by low fences, only about two duas high, so that the one zero asphalted areas could easily be distinguished from the grassy area in the middle.



The Principal had asked Alo to teach Bruce and Alice how to play the games.

"I wonder what kind of game they could have invented to play on these triangles?", said Bruce to Alice "perhaps it is a kind of croquet?"

"Croquet?", asked Alo, "What is that?"

Alice explained briefly to Alo how you played croquet.

"No", said Alo, "it's not like that. We use balls and rackets. The game is called tennis, but perhaps you don't play tennis in your country?"

"Of course we play tennis", rejoined Bruce, not a little cross with Alo for suggesting such a thing, "We play a lot of tennis, but we play it on rectangular courts, not on crooked ones like these!"

"Crooked?", replied Alo indignantly, "they are not crooked at all! Look how straight the fences are made that separate the inside from the three corners!"

"Please don't get cross", cried Alice "I am sure the lines are very straight! What Bruce means that at home the sides of our play areas are at right angles to each other"

"You know", added Bruce, "the areas are rectangles!"

"That's right", agreed Alice "they are rectangular courts."

"Sh-sh! Quiet! ", said Alo, putting his fingers across his lips, "in THREELAND we are not allowed to use that word!

"Of course, that's right!", agreed Bruce and Alice, "We had forgotten! The courts in our country have one one sides, and yours have one zero sides."

"You are getting really good", said Alo, "you are counting the sides the way we do!"

The children by this time had arrived at one of the triangular courts. Alo took a ball from a box as well as three rackets, he kept one for himself and gave one to Bruce and one to Alice. Each of the three corners was marked in white with one of the letters A, B or C. Alo took the area marked A, Bruce took B and Alice took C. Then Alo explained how you played.

"From your own court, if it is your serve, you try to get the ball into one of the other courts", said Alo "and the player who receives the ball has to play it back either to you or to the third player. Each time you hit the ball, you have to decide whether you are sending it to your right or to your left!"

"It sounds rather complicated!", said Alice "but I think I'll get the idea when we start playing"

"No, it's quite easy", said Alo, "you will see in a minute. At first we shall just hit the ball around without scoring, then I will explain the rules of the game and how you score"

So saying, he was as good as his word and served the ball. Of course, you had to hit the ball after it has bounced in your court, then you had to make it bounce in one of the other courts, so that the next player could have a bash at it with his or her racket. Alo sent the ball to Alice, who hit it over to Bruce's court, who in turn sent it back to Alo. Alo returned it to Bruce, who sent it over to Alice, who returned it to her brother, who then hit it over to Alo and so it went on. In the beginning Bruce and Alice dropped the ball or missed it several times, but they both thought it was fun to hit the ball around the big triangle.

"This is good fun!" exclaimed Alice "we ought to play tennis like this at home!"

"Well", said Alo, "while you are staying with us, you can play as much as you like. At home we have a court at the bottom of the garden behind the orchard, we can go there and play any time, after we have done our homework."

"You won't have to say it twice", said Alice, who was always very fond of all sorts of sporting activities, "But can you now explain about the service and how you score?"

"It's easy", said Alo, "Let me first explain about the service. The one who is in court A (which is me right now) plays a particular role. If the person in court A has served and the ball is dropped in your court, for instance by you missing it, then it is your service next."

"But surely that isn't fair!", objected Bruce.

"Just wait till I explain the whole thing", said Alo patiently.

"We are all ears!", said Bruce and Alice both at once.

"If B has served and the ball is dropped in A's court, it is again for B to serve next. But if it falls in C's court, then A must serve next. In the same way if C serves and the ball falls in A's court then C serves next, but if it falls in B's court, then A serves next."

"Yes", said Bruce, "but what happens when B serves and the ball drops in B's court?"

"Of course, I forgot about that one", said Alo, "If B serves and the ball drops in B's court then C serves next and of course if C serves and it drops in C's court, then B serves next. But if A serves and it drops in A's court, then A serves next. Let's have a try and see if you can tell each time who has to serve next."

There were one zero zero ways of serving the ball and having it dropped, as there are three possible servers and each serve could result in the ball dropping in any one of the three courts. These cases were divided evenly among the three players. The points were awarded as follows:

Ball drops in

		A	B	C
Served by	A	no score	A scores	A scores
	B	B scores	no score	B scores
	C	C scores	C scores	no score

If the ball is "out", namely if it falls in the central green area, then again the server gets the point.

A game is finished when one of the players obtains 1 0 0 points (don't forget that means nine points!). After one zero games, it is match. If each player has won a game, then it is a draw.

At the end of an hour's play they all felt tired but Bruce and Alice were both very excited that they had learned a new game. They decided to have lots more practice on the school courts as well as on the Kotos' court.

"How do they play tennis in TWOLAND", asked Bruce on the way to the school bus.

"It's nearly the same, except that there are two courts instead of three", replied Alo.

"It must be the same as the way we play it in our country then?, enquired Alice.

"Yes, when there are two players":, replied Alo "but the scoring is not the same. It is not the one who serves who gains the point, but the one who last hit the ball before it dropped. Even in THREELAND we sometimes play it like that, is that how you play it in your country?"

Yes, we do", replied Bruce "and I think it is fairer. Although since each player has the same number of serves, maybe it comes to the same thing. We are not used to your ways, that's all! Could we play TWOLAND tennis? Do you know anyone on the other side of the border where we could go and play?"

"Oh yes, and it isn't very far either", replied Alo "One of my friends over there has a tennis court in their garden, we could go there one day and play, if you like."

"Alo", whispered Alice "maybe I am risking going to prison, but I am dying to know how they play tennis in FOURLAND!"

"We had better wait before talking about that", said Alo, "you never know, somebody might be listening! Wait till we get to TWOLAND"

In TWOLAND you were allowed to talk about FOURLAND tennis, since four is one zero zero in TWOLAND even if it is one zero in FOURLAND! A group of the second order in TWOLAND is the same as a group of the first order in FOURLAND. But in THREELAND four was always referred to as one one and so it was not a very good number in THREELAND!

Having played a THREELAND tennis match in the Kotos' garden, Bruce asked if they could go over to TWOLAND and play another match there. So having obtained the Kotos' permission, Alo, Unta, Bruce and Alice walked over to the house of the Hecla family, whose house lay not far from the border, about one TWOLAND decima away. Alo introduced them to the Hecla children, whose names were Vic and Laura.

"Would you like to have a game of tennis?" asked Vic.

"Rather!", said Alice "It would be fun to play tennis in Fourland fashion, do you have a court where we can play in that way?"

"Oh yes", replied Vic,"last year we only had a one zero court, but we have since enlarged the courts. See over there ? Our court now is in the form of a square, divided into one zero zero smaller squares. Of course we don't need any lawn in the middle, as we use all the courts to play in."

As they were talking, they arrived at the tennis court, which was behind a clump of trees. Alice noticed that there were one zero zero zero trees in the clump, quite illegal in THREELAND. Had they been in THREELAND, there would surely have been one more tree in the clump, and they would be counted as one zero zero trees in Threelandish counting.

Vic then offered to explain about the courts.

"We used to have only two courts, they are the ones marked with the letters A and B. We have added two more and marked them with the letters C and D. Since you want to play FOURLAND tennis, we shall need all the courts, and I shall explain the rules: in fact there are two sets of rules, you can choose whether you want to follow one or the other set of rules. I will just start with the simple one"

"If it's been A's serve, then the next server is the one in whose court the ball falls. If a ball falls in the server's own court, then A has to serve next. If B or C or D has served, and the ball falls in one of the other two courts (out of B, C and D), then it is the third player out of B, C and D who serves next"

"For example, if B serves and the ball ends up in C's court, then D has to serve next"

"Oh, I see", said Alice "It's a kind of merry-go-round between B, C and D. Whichever two were involved in the serving and the dropping, the third one is serving next. It's a kind of triangle!"

"Yes", said Laura unhappily "but don't use that word here! We can use that word at your house, but we have to call it a polygon with one one sides!"

"There must be sixteen ways of serving and dropping the ball, or as you would say in TWOLAND, one zero zero zero zero ways" said Bruce.

"Or one two one ways, if we are speaking in Threelandish ways", added Alice.

"That's right", said Laura, "and each of the one zero zero players gets one zero zero of these possibilities, so it is quite fair!"

"When you get home tonight", said Vic "you can take a piece of paper and work it out by writing down all the possibilities. You could work out right away what would have to happen for A to serve next, couldn't you?"

"That's easy", said Alice "it's always when the ball drops in the server's court. So A will serve one zero zero times out of all the one zero zero zero zero possible combinations!"

"Great!", said Laura "you are getting pretty good at calculating in Twolandish!"

"I really like it now!", replied Alice "In the beginning I thought it was very muddling."

"Could you tell what must happen for D to serve next", asked Laura.

"Let's see", said Alice "I think I know!" One case is when A and D are involved in the serving and the dropping, the other is when B and C are involved in the serving and the dropping. The pattern (A, D) with (B, C) results in D serving next. It must be a similar pattern for B or for C serving next, right?"

"Yes, you are right, although I have never put it like that before", said Laura "But you can certainly play now, you seem to have molded the rules into easy patterns. Shall I explain the other game now?"

"Is the other game more difficult to remember?", asked Bruce.

"Actually, it is very similar to the rules in the first game", said Vic "but since they are so similar, sometimes we mix them up. If A serves, the rules are the same, namely the player in whose court the ball falls serves next. This is the same even in Fivelandish or Sixlandish tennis. If the ball falls in the court of the one who served, then the rule is different: In the case of B and D, the next server is C, but in the case of A and C, the next server is A. If B serves and it falls in D or if D serves and it falls in B, then A serves next, but if B serves and it falls in C or if C serves and it falls in B, then D serves next, as in the first game. Also if D serves and it falls in C or if C serves and it falls in D, then B serves next, as in the first game"

"That is really very confusing!' said Bruce "How do you manage to remember all these rules?"

"I keep a little table in my mind for each one", said Vic "like a little multiplication table. Look, here they are!"

SERVER	ball drops in A	B	C	D
A	A	B	C	D
B	B	$\boxed{A}$	D	$\boxed{C}$
C	C	D	A	D
D	D	$\boxed{C}$	B	$\boxed{A}$

SERVER	ball drops in A	B	C	D
A	A	B	C	D
B	B	$\boxed{C}$	D	$\boxed{A}$
C	C	D	A	D
D	D	$\boxed{A}$	B	$\boxed{C}$

The letters inside the tables show the next server. The first table gives the rules for the first game, the second table gives the rules for the second game. I have marked the places with smaller squares where the two games give you different next servers. There are only one zero zero cases out of one zero zero zero zero where you could make a mistake!"

"Don't you sometimes forget the rules, in spite of your little table?"asked Alice teasingly.

"Well", retorted Vic "If I do, I have another way of remembering who serves next. I just imagine A, B, C and D round a little circle, just as they are written on the courts. I can see that B is on the clockwise side of A, while D is on the counterclockwise side of A. So if the ball drops in B, the serve moves round one player in a clockwise sense. If it drops in D the serve moves round one player in the counterclockwise sense!"

"Yes, but what happens if it drops in A or in C? "enquired Alice.

"If it drops in A, it's easy", intervened Laura "the same person who served must serve again. If it drops in C, then the person diagonally opposite the last server has to serve next. Isn't that quite easy?"

"I think I've got the point", said Bruce "the whole thing seems to be about going round a circle, either clockwise or counterclockwise or half way round, right?"

Alice was not so sure about this game. The first one made a good pattern, but she could not, as yet, work out a good pattern for this second one. But they did play some games, using sometimes the first set of rules, sometimes the second set of rules. They played by "scoring" a point for anyone who dropped the ball. The lowest scorer was then the winner!

Vic asked them if they would like to play TWOLAND tennis, but they all decided that after the excitement of the FOURLAND games, it would not be very interesting to play TWOLAND tennis. Anyway it was getting dark and Unta thought they should be getting home. Mr. Hecla accompanied the children to the border post, and Mr. Koto made a joke of asking for their passports! This worried Alice at first, until she realized that Mr. Koto was just fooling!

"Tomorrow I believe you will have a lesson about how to tell the time" said Mr. Koto before letting them go through "I don't think you've had one yet!", he said turning to Alice.

"We can tell the time!", said Alice indignantly, "We are not babies!"

"But can you tell THREELAND time?", asked Mr. Koto.

"Is it different?", asked Alice.

"Well, time is time everywhere", replied Mr. Koto, "but you know that each State has its own money and measuring system, and as a matter of fact they also have their own time - system! So one hour in

TWOLAND is not the same as one our in THREELAND and it is certainly not the same as one hour in your own country!"

"Of course" said Bruce "but I would never have thought of it!"

"You would not have noticed", said Unta "since the hours in THREELAND are not so very different from your hours. We divide a day and a night into one zero zero zero equal intervals of time and call each such interval one hour. You divide yours into two two zero equal parts, don't you?"

Bruce and Alice thought about this as they sauntered over to the Kotos' house from the border post. When they saw Ata and she heard they were talking about the different hours, she added:

"You would have noticed the difference between your hour and a TWOLAND hour, they must be quite different. They divide their day and night into one two one equal parts, or into one zero zero zero zero parts, speaking in Twolandish ways"

"Do you mean that some hours are really longer than other hours?", asked Alice incredulously.

"You have two two zero hours in your day, and we have one zero zero zero hours in our day.", said Alo " The day and night we divide is always the same here or everywhere else, but if you divide it into more equal parts, you get shorter intervals of time for each part, don't you see?"

"I am not sure if I understand you", said Alice "Surely an hour is an hour, and must be the same everywhere. In all the countries in which we have traveled, the hours have always been the same! But you are right about the hours here, if they are a little shorter, I certainly have not noticed it"

"Id like to see a watch", said Alice, "then I'd know more what we are talking about."

"Here you are", said Unta, taking her watch off her wrist. It was a luminous watch and you could see the markings clearly on the clock face.

"Look here", objected Alice, "you said just now that you have one zero zero zero hours in your day. I can only see one zero zero on your watch!"

"Look at your own watch":, said Unta "are there twenty four markings on it? I can only see twelve!"

"Oh yes", said Alice "but the hour hand goes round twice."

"And ours goes round one zero times!", replied Alo.

"In THREELAND", put in Unta "instead of the hour hand going round twice, once from midnight to midday and then again from midday to midnight, it goes round once from midnight to one zero zero o'clock, then again once from one zero zero o'clock to two zero zero o'clock, and then again once from two zero zero o'clock to midnight"

"I suppose", added Bruce "the first round is the night, the second time is school time and the third time is family time!"

Yes" replied Unta "that is just how we think of it"

"I see", added Bruce "it is really a very sensible arrangement!"

"I don't care what you say" objected Alice "I much prefer to go by our own clocks, it seems to me much more sensible!"

"Not really", suggested Alo "You do everything else the TENLAND way, but you don't arrange your clocks the TENLAND way! You don't divide your day and night into ten equal parts! Alice, look at the numbers on the face of your watch: they are

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

and then all of a sudden you get back to 1. You should really have the numbers:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

around the clock face, would it not be more sensible? What is more you divide your hour into 2 0 2 0 equal parts which you call minutes. We learned about all this last month at school in Geography!"

"That is perfectly true", added Bruce, "Actually, on our train timetables we see things like trains leaving at 15.30, which means at 3.30 in the afternoon. It is more convenient, so you never confuse a morning train with an afternoon one. But you are right about the tens, we ought to have divided our day and night into ten equal parts, I beg your pardon, into one zero one parts!

"But if you wanted to be really logical" continued Bruce, "you should have divided your clock face into one zero zero zero parts, instead of which you have divided it into one zero zero parts."

"You win!" said Unta "Everything is not logical! Anyway, the way we have it, there are less markings on the clock face, imagine having one zero zero zero markings all round your clock face! That would be more confusing then ever!"

All this arguing had taken place outside the door of the Kotos' house, and when Mrs. Koto noticed them she said:

"Are you not getting a little tired? You've been playing tennis nearly all day, first at school, then here, then at your friend's house. It looks as though you might be stuck arguing outside the door for the rest of the evening! I wonder how you manage to stand up! Go on, children, the lot of you, get ready for bed."

"But Mrs. Koto!", objected Bruce, "we are not a bit tired, and your tennis is really wonderful, we've had a great time!"

"All right, then", agreed Mrs. Koto "Come and sit down in the family room and I'll bring you some hot chocolate and fruit"

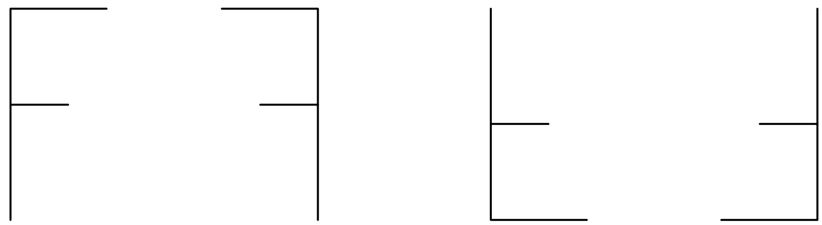
This seemed very acceptable to all the children and they all trooped into the family room.

"I've been thinking about that tennis game we were playing", said Alice to nobody in particular "It seems to me it is like flipping and turning something about!"

"I don't know what you mean", interjected Alo.

"Look, Alo", said Alice "Cut out a letter like a capital F. You can turn it over, kind of rolling it from left to right. Or you can roll it away from you. Or you can just turn it upside down. So you have four different F's. Look I'll draw them for you"

She took a sheet of paper and drew these four F's:



"Let us call the way we change the letter

(i) the right roll, (ii) the away roll and (iii) the turn around

Since we get from the proper F to B by doing a right roll, let us give the F at B the job of rolling any other letter to the right. Since we get from the proper F to C by an away roll, let us give the F at C the job of rolling any other letter away from you. And since we get from the proper F to D by turning it upside down, let us give the F at C the job of turning any other letter around"

"Why do you want to do all this turning around of letters when you were going to tell us about the tennis game?", asked Ata.

"I'll come to that in a minute", said Alice, "We also have to give A a job. Perhaps that should be to leave everything as it is!"

"So", continued Alice "doing an A to any letter, leaves that letter just as it is. And doing the job of any letter to A, will get us to that very letter. Is it not how the A court works in the tennis game?"

"And, doing the job of any letter to that same letter, will always give us the proper letter, namely A", added Alo enthusiastically. "Doing a B job to a B, or a C job to a C or a D job to a D, will always bring us back to A!"

"But wait!", said Alice " B-ing a C, or C-ing a B, will always give us a D! B-ing a D or D-ing a B will give us C, and of course D-ing a C or C-ing a D, will always get us a B! Again just the same as in the first game!"

"I am impressed!", said Bruce "You have always been good at drawing, because you can see how things look when you move them without actually moving them! But let me think of a way I could imitate our tennis game. Let's have a little quiet and think!"

The hot chocolate and fruit came in just then, so that gave Bruce some time to think. After about half an hour of hard thinking, Bruce said:

"I think I've got a different way. I usually think with numbers, just as you Alice think in pictures. So here are my numbers:

1,   3,   5,   7

No other numbers are allowed. If you multiply any two of these numbers, and then you take away from the product as many 8's as you can, you always get one of the four numbers. It is obvious that 1 behaves like the A court, or like Alice's normal letter F"

"And each one multiplied by itself gives us 1!" cried Alo "I've just worked it out. Look

3 x 3 = 9 but 9 - 8 = 1

5 x 5 = 25, but 25 - 3 x 8 = 1

7 x 7 = 49, but 49 - 6 x 8 = 1

so as long you take all the 8's away, we always get 1! And that is what happens in the tennis game if the ball drops in the server's court!"

"But wait!", insisted Bruce, "we have the three way merry-go-round as well, because

3 x 5 = 15 but 15 - 8 = 7

and 3 x 7 = 21 but 21 - 2 x 8 = 5

and 5 x 7 = 35 but 35 - 4 x 8 = 3

So it all works just like the game and just like Alice's letters!"

"That's really beautiful!" said all the Koto children admiringly. "But how would you do the second game with numbers?", asked Alo.

"I think I can do it with four other numbers, but this time we shall have to take away 5's, instead of 8's. I think it'll work, let me see. Let A have the number 1 again, I suppose B could have the number 2, then we'll give C the number 4, since we are multiplying round the circle, that leaves the number 3 for D"

"There is something wrong with the 3, I think", said Alice, "I can see that 1 x 2 = 2, and that 2 x 2 = 4, but 2 x 4 is not equal to 3!"

"Yes, but Alice, don't you see?", interrupted Bruce " If you take away 5 from 8 you do get 3! So multiplying by 2, we DO go round the circle

A = 1, B = 2, C = 4, D = 3, and back to A = 1

because 3 x 2 = 6 and 6 - 5 = 1 !

"So that is your clockwise turn round the courts" said Alo, "but where is the counterclockwise one?"

"You just multiply by 3, and as long as you take away 5 every time you get a number bigger than 4, you get the counterclockwise circular tour of the courts! Look

1 : 1 x 3 = 3 : 3 x 3 = 9, but 9 - 5 = 4:

then 4 x 3 = 12 but 12 - 2 x 5 = 2, and finally

2 x 3 = 6 but 6 - 5 = 1 so we are back to 1!"

"And I suppose the number 4 will give you the diagonal change of server, shifting between A and C and between B and D." suggested Alice "Let's see:

1 x 4 = 4, but also

4 x 4 = 16 but 16 - 3 x 5 = 1.

So 1 and 4 are exchanged. In the same way

$2 \times 4 = 8$ but $8 - 5 = 3$

and $3 \times 4 = 12$ but $12 - 2 \times 5 = 2$

so 2 and 3 are also exchanged! So we can imagine the courts marked like this " and Alice drew this simple diagram:

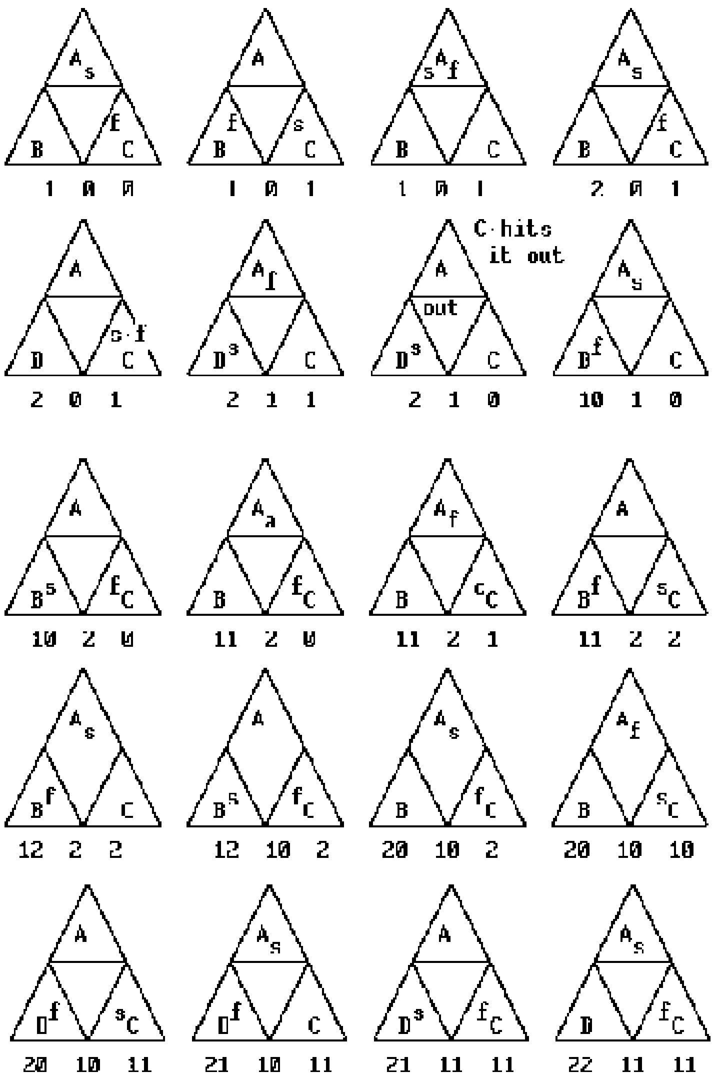
1	2
3	4

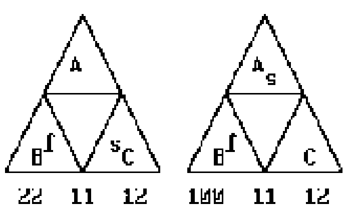
A	B
D	C

After all this Mrs. Koto hounded them all off to bed!

THINGS TO DO AND THINGS TO THINK ABOUT.

(1) Here is a sketch of a THREELAND tennis match. The letter s means the server, the f means where the ball fell. The numbers show the score at each stage of the game.





When the ball falls in the server's court, there is no score. The one who hits it "out" loses a point.

This game is won by player A.

Make a court and play the game!

(2) Take a piece of square cardboard and draw a FOURLAND tennis court on it. Mark the squares A, B, C and D in the clockwise sense in that order.

Make a die by making a small cube, and write on the faces:

A on one face, B on another, C on another and D on another, OUT on the fifth face and OUT on the sixth face.

Use a counter to represent the ball

Use the die to determine where the ball falls.

Do the scoring in Fourlandish numbers.

(3) Make a Threelandish tennis court out of cardboard, in the form of an equilateral triangle, divided into four equilateral triangles. Color the central triangle green to represent the lawn there. Mark the triangles at the corners A, B, and C respectively.

Make a die, and write A, B and C one on each of three faces. On the remaining three faces write OUT.

Use a counter to represent a ball.

Use the die to determine where the ball falls.

Do the scoring in Threelandish numbers.

(4) In a room, there are two lamps, lamp A and lamp B. There are four switches at the door. One turns lamp A on or off, another turns lamp B on or off. The third one works both lamps at the same time. The fourth one has its fuse broken and does not do anything.

Call the switches X, Y, Z and F.

Somebody comes in and uses one of the switches, then one of the others or may be the same one again. Is there one switch that would have had the same effect on the lamps? If so, which one?

Try to solve this problem for a number of pairs of switches.

Does this problem remind you of one of the tennis games? If so, which one?

(5) Bruce, Alice and Ata play the following counting game:

Ata says 1, Bruce says 2, Alice says 1 0, Ata says 1 1, Bruce says 1 2 and they go on counting like this till they reach the number 1 0 0 0 0.

Choose a number written in Threelandish, and try to guess which child must have said that number. Is there a quick way to tell?

Add an Ata number and an Alice number. Whose number do you get? Do you always get the same person's number if you add ANY Ata number to ANY Alice number?

Are the rules for adding Ata's, Alice's and Bruce's numbers at all like the rules for playing Threelandish tennis?

See if you can find a way in which the two games are alike.

(6) Now let us pretend that Bruce, Alice, Ata, Unta and Alo have gone over to FIVELAND, and are playing the FIVELAND counting game, with the five of them standing round a circle in some such way as this:

```
          Ata
        /      \
     Alo        Unta
      \           /
      Bruce -- Alice
```

Ata starts counting from 0, and they go round clockwise, each child saying the next number, but in the Fivelandish way.

Can you easily tell which number belongs to which child?

Work out whose numbers you get when you add different children's numbers, also what you get if you add two numbers belonging to the same child.

If Unta is the 1-child, Alice the 2-child, Bruce the 3-child and Alo the 4-child, by what number would you have to go on multiplying so as to get the children

Unta ---> Alice --> Alo --> Bruce --> Unta --> ....

in the above order?

By what number would you have to multiply, so as to get the reverse order?

Invent a similar game in SEVENLANDISH for seven children.

By multiplying, what kind of sequences of children can you get? You should be able to find some cycles of six, some of three and some of two!

(7) Imagine you are in THREELAND and that you have been given a large thin equilateral triangle. The teacher has asked you to hold the triangle firmly in one position with one hand in the sunlight, and to cast its shadow on to a painting easel so you can draw its shadow. You can change the angle of the easel.

(i) Will the shadow always be a triangle?

(ii) Can the shadow be equilateral? If so, how would you make sure that it was?

(iii) Can you make two sides of the shadow triangle equal, but the third one a different length?

(iv) Can you make your shadow triangle have one right-angle?

Before you try to answer the questions, make sure you have moved the easel around as well as your triangle!

(8) Now imagine that you are in FOURLAND and that you are looking at the shadows of a square.

(i) Will your shadow always have four sides?

(ii) Will opposite sides of your shadow be parallel?

(iii) Can you make the shadow to be a rectangle that is not a square? Or can you make it into a square?

If you can use a slide projector, try your shadows in the light of the projector. The answers to the three questions, will they be the same, or will they be different?

(9) Bruce and Alice were given the following problem when they were visiting a school in FOURLAND:

They were in a square room, where one wall was to the North, one to the East, one to the South and one to the West. The floor was tiled with squares. Bruce and Alice always had to stand inside one of the squares, unless they were walking. Bruce could walk as many steps as he liked, but only towards one of the walls. Every time Bruce took a step towards one of the walls, Alice had to take a step along one of the diagonals, and she had to do it according to these rules:

Bruce 1 step North ---> Alice 1 step North-East

Bruce 1 step South ---> Alice 1 step South-West

Bruce 1 step East ---> Alice 1 step South-East

Bruce 1 step West ---> Alice 1 step North-West

If Bruce describes a square, what figure does Alice describe? If Bruce does it clockwise, does Alice do so as well?

How much more floor space does Alice's walk enclose than Bruce's?