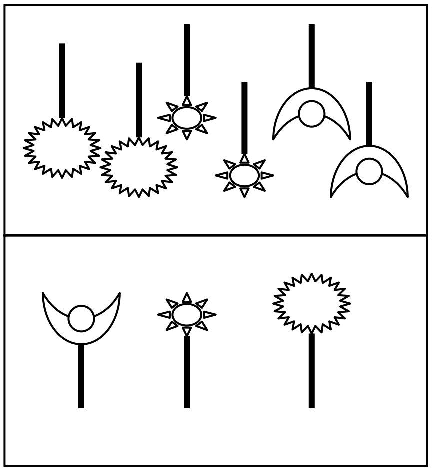
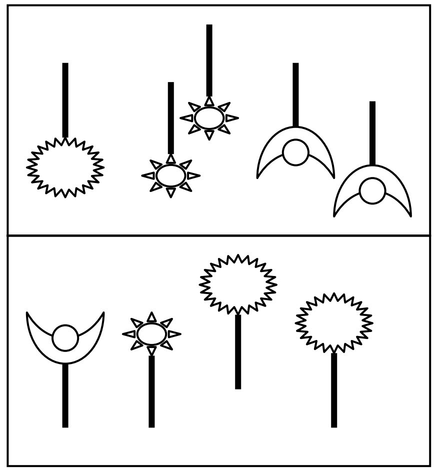


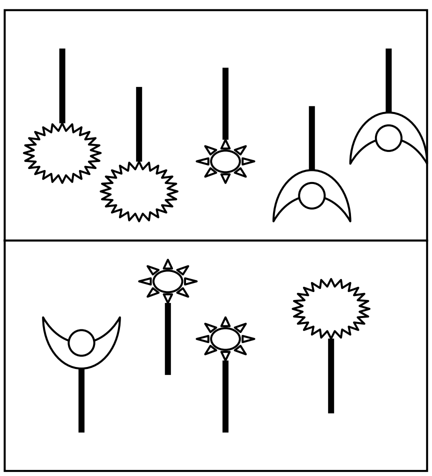
The twenty-six pictures game.

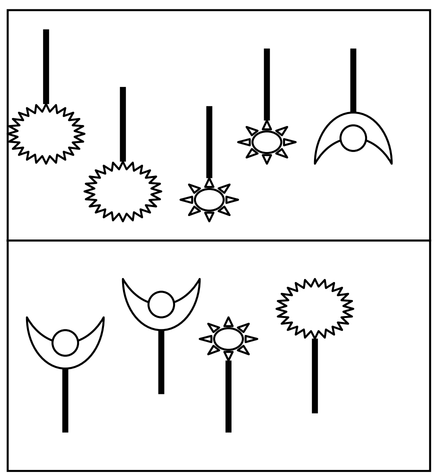
1. Instructions of how to make our "toys".

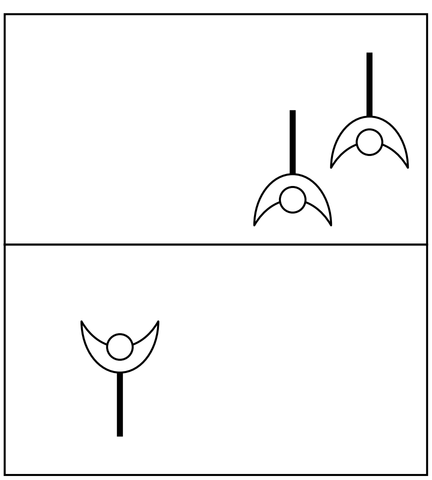
Cut out these "double" pictures and fold each one at the dividing line between the two pictures. You can then stand them up so that each picture appears the right way up. You will have thirteen such "double pictures"; these will be our "playthings" for the following activities..

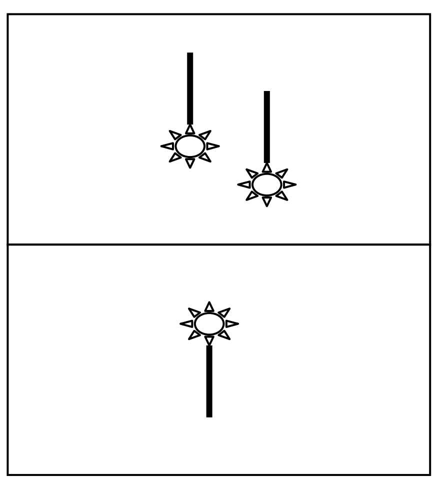


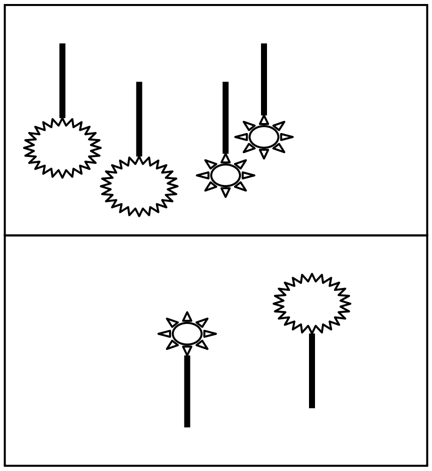


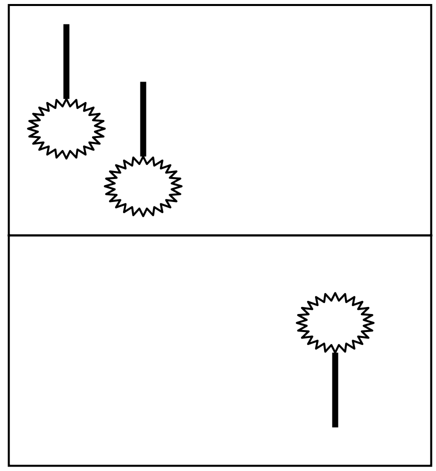


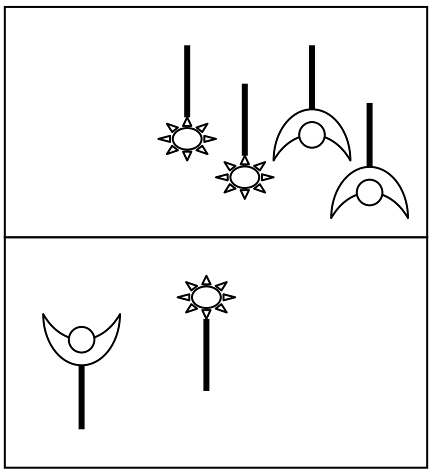


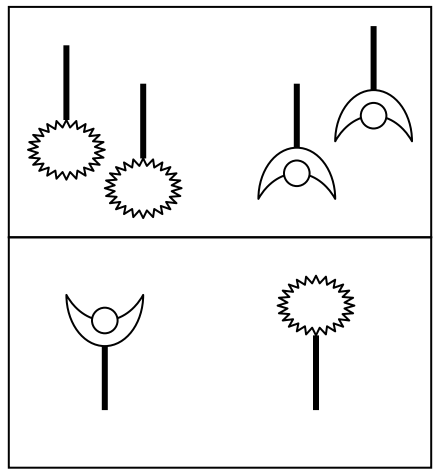


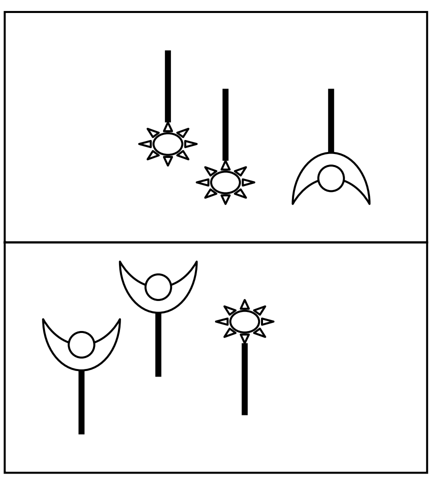


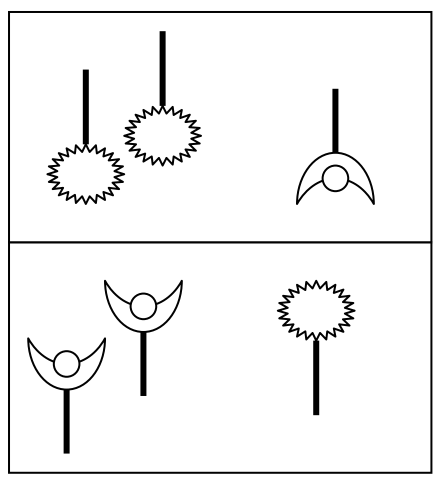


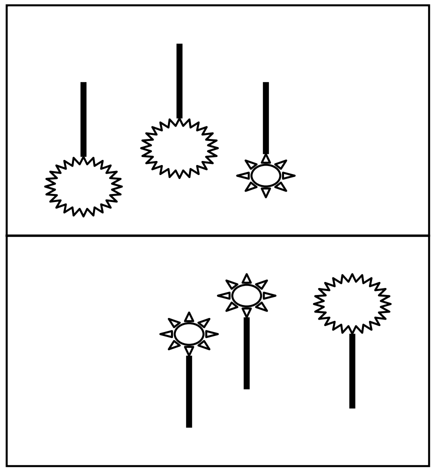












You will have noticed, if you have looked through the pictures, that there are thirteen pictures the right way up and another thirteen upside down. This is so that if you cut out each double picture, you can fold along the line separating them, so as by standing them up, both pictures will appear the right way up.

I shall call the big round flowers sunflowers, the small round ones daisies. I think the third kind look quite like tulips!

You will also no doubt have noticed that in each set of two pictures, the numbers 2 and 1 are interchanged.

I shall regard a total of three of any kind of flower as zero. So the total "number" of flowers on any set of two pictures that have been drawn together is then zero. In fact the only "numbers" that will be "allowed" in our games are: zero, one and two.

2. **Addition and multiplication**.

Addition of our three numbers is done almost like the normal addition, except that

$$\mathbf{2 + 2 = 1}$$

because out of the four we would normally get as our sum, three are counted as zero, so instead of four, which is not one of our three numbers, we have the number one.

Multiplication is also almost the same as in "normal" multiplication, with the exception of two times two, and again we have, for the same reasons as before:

$$\mathbf{2 \times 2 = 1}$$

I shall describe each picture simply by stating how many of each kind of flower can be found on the picture. I shall "count" the tulips first, then the daisies and then the sunflowers. For example [1, 1, 2] will mean that there is one tulip, one daisy and two sunflowers in the picture; or [2, 0, 2] will mean that there are two tulips, no daisies and two sunflowers in the picture.

3. Adding pictures.

We can "add" a picture to another picture by simply adding tulips to tulips, daisies to daisies and sunflowers to sunflowers.

Let me call each double picture, each "toy", a PRISM. You could actually make the "toys" look more like prisms by making triangular prisms out of cardboard and sticking the double pictures on two of the faces of the cardboard prism. You will be using the third face of the prism as its base on which it can stand.

If you have cut out and suitably bent your "toys" into prisms so you have all the pictures the right way up, you can play the following "game":

(i) Pick any two of your prisms.
(ii) Choose one picture on one of the prisms and another picture on the other.
(iii) Add these chosen pictures.
(iv) Find the prism which has this picture on it.
(v) We say that you have "captured" this third prism.
(vi) Place it with your two chosen prisms, thus making an enlarged set.
(vii) Now choose any two pictures from prisms in your enlarged set
(viii) Add these pictures and find the prism with it on and so capture it.
(ix) Go on like this capturing prisms by adding more and more pictures.
(x) When you have four prisms in your set, any further sums will be in this set.
(xi) We can call such a set of four prisms a family of prisms.

Try to see in what way the eight pictures on your four prisms are alike.
The way they are alike must be such that none of the pictures outside your set is like them. Such a "property" can then be used as the name of the family.

For example if neither of your first prisms had any sunflowers, then there is no way you can capture any prisms with sunflowers on it. So such a family could be called the "No sunflowers family".
You might have chosen both prisms so that in all four pictures the number of daisies is equal to the number of tulips, In adding your pictures, you would only get other "sum pictures" with the same number of daisies as tulips. You could call such a family the "Daisies same as tulips family"

There are actually thirteen families you can find by starting with two given prisms. Try to find as many as possible of these families. Give each family a suitable name. Do not forget that the name must apply to all members of the family, but must not apply to any pictures outside the family.

Here is an example. Suppose you choose the prisms:

1 2 1
2 1 2

1 2 0
2 1 0

Let us find the sum 1 2 1 + 1 2 0 = 2 1 1, so we capture the prism

2 1 1
1 2 2

But then what about 1 2 1 + 2 1 0 = 0 0 1 ? We have captured

0 0 1
0 0 2

If we try any more adding of pictures we already have, we shall again get one of the pictures we already have. So we have a family.

What could be the name of this family?

Except for the last prism captured, we always have a different number of tulips and daisies. If you look in the next section, you will find such a family listed at the 7th place. We could make the strange convention that zero is equal to zero as well as different from zero, in which case all four prisms have the property of having a different number of tulips from the number of daisies.

If you don't like such an absurd convention (and you would be in good company!), you will need to change some names. For example we could say that in this family, if we add the number of tulips to the number of daisies, we always get zero. Maybe it is not quite so absurd to call the numbers 3 and 0 equal as to call zero the same as itself as well as different from itself! You can make your choice, or maybe you could find an even more suitable name!

4. Suitable names for the thirteen families.

Here are some suggested names for the families that you can find. You are welcome to improve on the names.

(1) No tulips, (2) No daisies, (3) No sunflowers,

(4) Tulips same as daisies,

(5) Daisies same as sunflowers,

(6) Tulips same as sunflowers

(7) Tulips different from daisies,

(8) Daisies different from sunflowers

(9) Tulips different from sunflowers

(10) Tulips plus daisies equals sunflowers

(11) Daisies plus sunflowers equals tulips

(12) Daisies plus sunflowers equals tulips

(13) Total number of flowers zero

5. Make a three by three pattern with nine prisms.

Here is a fun problem you might like to solve. I give a solution latrer on, but don't look if you want the fun of finding out for yourself.

(i) Choose nine of your prisms.
(ii) Place them in three rows of three prisms.
(iii) Make sure, however, that prisms in the same row all belong to one family.
(iv) Also make sure that prisms in the same column all belong to one family.
(v) For good measure, make it so for the diagonals as well!

One useful hint would be to choose your nine prisms in such a way that the four prisms not being used should all belong to one family. If you are not careful with this, you will have difficulties!

What is the fourth member of each row-family?
Of each column family? Of the diagonal families?

Are the "missing members" all in the family outside your pattern?

With the "outside family" you will have used nine families altogether: three row-families, three column-families, two diagonal families and the outside family. There are four more families! Where are they in your pattern?

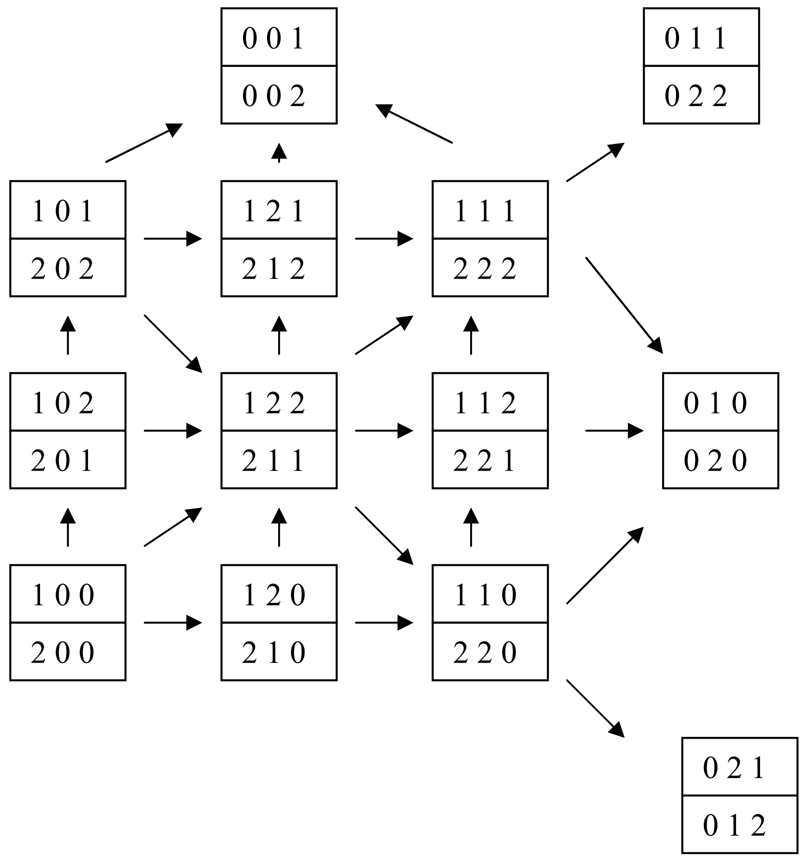
Given any two families, do they always have a common member?

Given two prisms, is there just one family to which they both belong?

To how many families does each prism belong?

If you had a picture with no flowers on it at all, would it belong to any of the families? If so, to which ones would it belong?

Here is a solution to the three by three pattern problem:



The four prisms "left out" of the pattern form the family with no tulips. These prisms are:

(i) only sunflowers,
(ii) daisies same as sunflowers
(iii) only daisies,
(iv) daisies and sunflowers equal zero

The first of these is a member of each of the three column-families, the second one completes one of the diagonal families, the third one is a member of each of the three row-families and the fourth one completes the other diagonal family.

The four "missing" families are along the "sub-diagonals" with the prism in the opposite corner in each case. The names of these families are:

(a) tulips plus sunflowers equal daisies
(b) daisies plus sunflowers equal zero
(c) tulips plus daisies equal sunflowers
(d) total number of flowers equal zero

6. **Bundles.**

Every prism belongs to four families, and every family consists of four prisms. Any two families have one prism in common, and any two prisms belong to just one family.

Four families all having a prism in common form a BUNDLE.

A “bundle game” could be played like this:

(i) All thirteen family names and all thirteen prisms are placed randomly on the table.
(ii) One player picks up three names at random.
(iii) Players have to determine whether these names belong to a bundle or not.
(iv) The first player who is sure will shout either “BUNDLE!” or “NO BUNDLE”.
(v) A mistaken declaration can be challenged by the other players.

Naturally the game can also be played by a player picking three prisms at random, the other players having to decide whether the three prisms do or do not all belong to one and the same family. The “shouts” then would be “FAMILY!” or else “NO FAMILY”.

An interesting bundle problem is the following:

(i) Place all the prisms in a tidy row in front of you.
(ii) By each prism place a name, not necessarily a name belonging to that prism.
(iii) Change the positions of the prisms and/or of the names so that any three prisms belonging to one and the same family, should have by them three names that are a part of a bundle.

Don’t look, but here is one possible interesting solution:

0 0 1	0 1 0	1 0 0	1 1 0	1 0 1	0 1 1
0 0 2	0 2 0	2 0 0	2 2 0	2 0 2	0 2 2
S=0	D=0	T=0	T+D=0	T+S=0	D+S=0

1 2 0	1 0 2	0 1 2	1 1 2	1 2 1	2 1 1
2 1 0	2 0 1	0 2 1	2 2 1	2 1 2	1 2 2
T=D	T=S	D=S	T+D=S	T+S=D	D+S=T

1 1 1	
2 2 2	$\longrightarrow$ T+D+S=0

7. Making up rules for sequences of pictures.

It is quite easy to make up simple rules through the use of which the prism coming after any given prism can be determined. Here is one suggestion:

The picture you are looking at		**the next picture**
Tulips plus daisies	**will give**	**Tulips**
Tulips plus sunflowers	**will give**	**Daisies**
Tulips	**will give**	**Sunflowers**

Do not forget our addition rules! From any number you get by adding which is greater than two, you have to subtract three to get the "sum"!

This is how the sequence looks like:

[0 0 1], [0 1 0], [1 0 0], [1 1 1], [2 2 1], [1 0 2], [1 0 1], [1 2 1], [0 2 1]

[2 1 0], [0 2 2], [2 2 0], [1 2 2] and back to [0 0 1] again.

You will notice that only one side of each prism occurs in the sequence.
If you started with [0 0 2] instead of with [0 0 1], you would get all the other sides, but of course none of the above.
It is interesting to look for those pictures here that belong to one and the same family.
Look at any two pictures with just one picture between them. After the second one of these "jump over" two pictures and you will get two more straight after the "jump" that belong to the family to which your first two belong. In other words in any run of seven pictures you will have the pattern:

Yes No Yes No No Yes Yes

Where Yes means "belongs to a given family" and No means "does not belong to this given family". For example for the "No daisies family" you will need to look at the first seven pictures, the first, the third, the sixth and the seventh pictures belonging to this family, while no other picture in the sequence does so.

You might also have noticed that if you add any three pictures in a row, you always get the fourth picture.

Try to make up your own sequences, inventing different rules and see how the families are placed within your sequences.

Here is another set of rules. These rules generate a sequence taking in all the pictures:

The word "behind" will mean look on the other side ("behind") of the prism and see how many flowers there are there of the kind being referred to.

The picture you are looking at		The next picture
Tulips behind plus daisies	**will give**	**tulips next**
Daisies plus sunflowers behind	**will give**	**daisies next**
Tulips plus daisies	**will give**	**sunflowers next**

You will find that the rules take you through the thirteen prisms, then the sequence returns to the back of the first prism with which you started and goes through the back of all the prisms, in the same order.

You might also find that the rule [yes, no, yes, no, no, yes] also works in this case.

Try to find other rules that take you through all the pictures and see in what order the members of any one family come up as you go through the sequence.

For more mathematical games, see the web site at http://www.zoltandienes.com